Exercise 18

Let L_n denote the left-endpoint sum using n subintervals and let R_n denote the corresponding right-endpoint sum. In the following exercises, compute the indicated left and right sums for the given functions on the indicated interval.

$$R_4 \text{ for } x^2 - 2x + 1 \text{ on } [0, 2]$$

Solution

Since we're using the right-endpoint sum with $n = 4$ to approximate the integral of $x^2 - 2x + 1$ from 0 to 2, the sum is taken from 1 to 4 rather than 0 to 3.

$$\begin{aligned}
\int_0^2 (x^2 - 2x + 1)\,dx \approx \sum_{i=1}^{4} (x_i^2 - 2x_i + 1)\Delta x &= \sum_{i=1}^{4} [(0 + i\Delta x)^2 - 2(0 + i\Delta x) + 1]\Delta x \\
&= \sum_{i=1}^{4} [i^2(\Delta x)^2 - 2i\Delta x + 1]\Delta x \\
&= \sum_{i=1}^{4} \left[i^2 \left(\frac{2-0}{4} \right)^2 - 2i \left(\frac{2-0}{4} \right) + 1 \right] \left(\frac{2-0}{4} \right) \\
&= \sum_{i=1}^{4} \left[i^2 \left(\frac{1}{2} \right)^2 - 2i \left(\frac{1}{2} \right) + 1 \right] \left(\frac{1}{2} \right) \\
&= \sum_{i=1}^{4} \left[i^2 \left(\frac{1}{2} \right)^3 - 2i \left(\frac{1}{2} \right)^2 + \left(\frac{1}{2} \right) \right] \\
&= \sum_{i=1}^{4} \left[i^2 \left(\frac{1}{8} \right) - 2i \left(\frac{1}{4} \right) + \left(\frac{1}{2} \right) \right] \\
&= \sum_{i=1}^{4} i^2 \left(\frac{1}{8} \right) - \sum_{i=1}^{4} i \left(\frac{1}{2} \right) + \sum_{i=1}^{4} \left(\frac{1}{2} \right) \\
&= \frac{1}{8} \sum_{i=1}^{4} i^2 - \frac{1}{2} \sum_{i=1}^{4} i + \frac{1}{2} \sum_{i=1}^{4} 1 \\
&= \frac{1}{8} \left[\frac{4(4+1)(8+1)}{6} \right] - \frac{1}{2} \left[\frac{4(4+1)}{2} \right] + \frac{1}{2}(4 \cdot 1) \\
&= \frac{1}{8}(30) - \frac{1}{2}(10) + \frac{1}{2}(4) \\
&= \frac{3}{4}
\end{aligned}$$